Mathematics-1 Sets and Functions

Topics : Computer engineering
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"Sets and Functions" is a fundamental topic in mathematics that serves as the building block for many other mathematical concepts. Here's an overview of what this chapter typically covers:

1. **Introduction to Sets:**
    - Definition of a set: A collection of distinct objects.
    - Representation of sets: Roster notation, set-builder notation.
    - Cardinality of sets: Number of elements in a set.
    - Subsets and supersets: Relationship between sets where every element of one set is also an element of another set (subset) or vice versa (superset).
    - Universal set: Set containing all the objects under consideration.
2. **Types of Sets:**
    - Finite set: A set with a countable number of elements.
    - Infinite set: A set with an uncountable number of elements.
    - Empty set (null set): A set with no elements.
    - Singleton set: A set with exactly one element.
    - Equal sets: Sets having exactly the same elements.
    - Power set: Set of all subsets of a given set.
3. **Operations on Sets:**
    - Union of sets: Combination of all elements from two or more sets.
    - Intersection of sets: Elements common to all sets.
    - Difference of sets: Elements present in one set but not in another.
    - Complement of a set: Elements not belonging to the set within a universal set.
    - Cartesian product: Set of all ordered pairs from two sets.
4. **Venn Diagrams:**
    - Graphical representation of sets using circles or rectangles.
    - Illustration of set operations (union, intersection, difference, complement) using Venn diagrams.
5. **Functions:**
    - Definition of a function: A relation between two sets where each input has exactly one output.

- Domain and range: Set of all possible inputs and outputs of a function, respectively.
- Types of functions: One-to-one (injective), onto (surjective), and bijective functions.
- Composite functions: Combination of two or more functions.
- Inverse functions: Function that "undoes" the original function.

6. **Special Functions:**
   - Identity function: Function where the output is equal to the input.
   - Constant function: Function where the output is the same constant value for all inputs.
   - Polynomial function: Function defined by a polynomial expression.

Sure, let's illustrate "Sets and Functions" with examples:

Sets:

- **Finite Set Example:**
  - Let's consider a set A = {1, 2, 3, 4}. This is a finite set because it has a countable number of elements.
  - Another example of a finite set could be the set of weekdays: B = {Monday, Tuesday, Wednesday, Thursday, Friday}.
- **Infinite Set Example:**
  - The set of natural numbers: N = {1, 2, 3, ...} is an infinite set because it continues indefinitely.
  - Similarly, the set of real numbers is also infinite.
- **Empty Set Example:**
  - Let's denote the empty set as Ø or {}. It is a set with no elements.
  - For example, the set of even numbers that are odd: C = {x | x is an even number and x is odd} = Ø.
- **Union and Intersection Example:**
  - Let A = {1, 2, 3} and B = {3, 4, 5}. The union of A and B is A ∪ B = {1, 2, 3, 4, 5}.
  - The intersection of A and B is A ∩ B = {3}.
- **Complement Example:**
  - Let's consider a universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. If A = {2, 4, 6, 8}, then the complement of A is A' = {1, 3, 5, 7, 9, 10}.
- **Functions:**
- **One-to-One (Injective) Function Example:**
  - Let's define a function f: A → B where A = {1, 2, 3} and B = {a, b, c} such that f(1) = a, f(2) = b, and f(3) = c.
  - This function is one-to-one because each element in A maps to a unique element in

B.

- **Onto (Surjective) Function Example:**
  - Consider a function g: A → B where A = {1, 2, 3} and B = {a, b}. Let g(1) = a, g(2) = b, and g(3) = b.
  - This function is onto because every element in B is mapped to by at least one element in A.
- **Composite Function Example:**
  - Suppose we have two functions f: X → Y and g: Y → Z. Let f(x) = x^2 and g(y) = 2y.
  - The composite function h(x) = g(f(x)) = 2(x^2).
- **Inverse Function Example:**
  - Let's define a function f: A → A where A = {1, 2, 3, 4} such that f(1) = 2, f(2) = 3, f(3) = 4, and f(4) = 1.
  - The inverse function of f, denoted as f^(-1), would be such that f^(-1)(2) = 1, f^(-1)(3) = 2, f^(-1)(4) = 3, and f^(-1)(1) = 4.