Mathematics 1 - Integration

Topics : Computer engineering
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1. **Definition:**
    - Integration is the process of finding the integral of a function. It is the reverse operation of differentiation.
    - The result of integration is called the antiderivative or indefinite integral of the function.
2. **Integral Notation:**
    - The integral of a function f(x) with respect to x is denoted by ∫f(x) dx.
    - The symbol ∫ represents integration, f(x) is the integrand, and dx indicates the variable of integration.
3. **Definite Integral:**
    - A definite integral represents the area under the curve of a function between two specified limits of integration.
    - It is denoted by ∫[a, b] f(x) dx, where a and b are the lower and upper limits of integration, respectively.
4. **Fundamental Theorem of Calculus:**
    - The Fundamental Theorem of Calculus establishes a connection between differentiation and integration.
    - Part I states that if F(x) is an antiderivative of f(x), then ∫[a, b] f(x) dx = F(b) - F(a).
    - Part II states that if f(x) is continuous on an interval [a, b], then F(x) = ∫[a, x] f(t) dt is an antiderivative of f(x).
5. **Integration Techniques:**
    - Substitution: Also known as the u-substitution method, it involves substituting a new variable to simplify the integrand.
    - Integration by Parts: A technique based on the product rule for differentiation that allows us to integrate products of functions.
    - Partial Fractions: Used to decompose rational functions into simpler fractions for integration.
    - Trigonometric Integrals: Involves applying trigonometric identities to integrate trigonometric functions.
    - Improper Integrals: Integrals with infinite limits or integrals with discontinuous integrands.

6. **Applications of Integration:**
    - Integration is used to find areas, volumes, arc lengths, surface areas, and various physical quantities in real-world applications.
    - It is essential in physics, engineering, economics, and other fields for solving optimization problems and modeling continuous processes.